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Technical Notes

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Nonlinear Interactions Between Forced and Self-Excited Acoustic Oscillations in Premixed Combustor

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Introduction

THIS note describes an investigation of the nonlinear interactions between natural acoustic modes and forced oscillations in an unstable combustor. Such instabilities are a serious problem in low emissions, premixed combustion systems [1,2]. This work is motivated by two prior sets of observations. First, there has been increased interest in the development of control systems which can eliminate these instabilities [2–4]. Typically, active control schemes harmonically modulate the fuel at the same frequency, but out of phase, with the instability. In other cases, the use of open-loop excitation, at frequencies not necessarily equal to that of the instability, has been successfully employed; e.g., see Lubarsky et al. [4] or Jones et al. [5].

Second, this work is motivated by measurements of the response of flames to forced oscillations which have reported nonlinear interactions between a natural combustor mode and those due to external forcing [6]. Specifically, a steady decrease in the amplitude of the unstable mode was observed as the driven mode amplitude increased. This behavior was attributed to frequency locking, a well-known nonlinear oscillator phenomenon [7]. Frequency locking is due to nonlinear interactions between oscillations at separate frequencies and results in a decrease in amplitude of the self-excited or natural mode oscillations as the amplitude of the driven oscillations increases.

The basic phenomenon of frequency locking can be illustrated from a model equation for a nonlinear, second-order oscillator:

$$\ddot{x} + \omega_0^2 x = \varepsilon\left(\dot{x} - \frac{1}{3}\dot{x}^3\right) + E(t) \tag{1}$$

where ε is a small parameter and $E(t) = K\cos(\Omega t)$ is the external forcing. The solution of this equation to leading order in ε is a superposition of the natural and forced oscillations and takes the form [7]:

$$x = a(t)\cos(\omega_0 t + \beta) + \frac{K}{\omega_0^2 - \Omega^2}\cos\Omega t + O(\varepsilon) \tag{2}$$

Following Nayfeh and Mook [7], the amplitude of the response $a(t)$ can be determined analytically:

$$a^2 = \frac{4\eta}{\omega_0^2 + \left[\left(4\eta/a_0^2\right) - \omega_0^2\right]\exp(-\varepsilon\eta t)} \tag{3}$$

where a_0 is the initial amplitude of oscillation and $\eta = 1 - \frac{1}{2}\Omega^2 K^2(\omega_0^2 - \Omega^2)^{-2}$. Figure 1 plots the steady-state solution $(t \to \infty)$ of the self-excited amplitude for increasing forcing amplitude. It shows the monotonic reduction in instability amplitude $a(t)$ until its amplitude reaches zero, as the forcing amplitude $E(t)$ is increased. Similar behavior is observed experimentally, as described in the rest of the note.

The objective of the present study is to perform a systematic study of this frequency-locking phenomenon by examining the role of frequency spacing between the driven and self-excited oscillations, amplitude of excitation, and operating condition on the natural mode response. In particular, effort was made to find conditions where open-loop forcing was effective, but also where it was not. For the sake of space, only representative results for two conditions are shown, but the reader is referred to the lead author's thesis for the full set of data [8].

Experimental Setup

Experiments were performed with an atmospheric, 10 kW swirl-stabilized burner, described in Bellows et al. [9]. To ensure that acoustic oscillations do not affect fuel/air mixing processes, the air and fuel are introduced upstream of a choke point. The mixture goes through the nozzle, consisting of a 40 deg swirler and an annular passage, and then expands into a cylindrical 70 mm i.d. quartz tube combustion chamber. The instability frequency can be changed by simply switching out quartz tube lengths or changing the thermal power setting of the combustor. For this study, tube lengths of 305 and 406 mm were used. Pressure oscillations are measured with two pressure transducers mounted downstream of the swirl vanes, 5.85 and 7 cm upstream of the rapid expansion. Velocity oscillations are calculated using the two-microphone method. Oscillations are driven in the combustor by two loudspeakers mounted into the inlet section. The two operating conditions investigated in this study are shown in Table 1.

Experimental Results and Discussion

Condition 1 tests were performed at a nominally unstable condition, with an instability frequency of $f_{\mathrm{ins}} = 461$ Hz. The forcing frequencies investigated ranged from $f_{\mathrm{drive}} = 150$–430 Hz and, for all cases, the overall acoustic power was substantially reduced by the presence of acoustic forcing. A typical result is shown in Fig. 2. The nominal amplitude of the 461 Hz instability is about 1.5% of the mean pressure in the combustor. In this particular case, forced oscillations are excited at 200 Hz over a range of amplitudes. As shown in Fig. 2a, increased forcing levels cause the 461 Hz mode amplitude to decrease, and to nearly disappear at high driving

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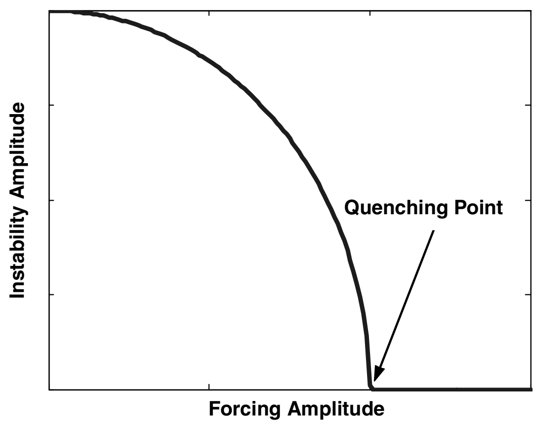


Fig. 1 Theoretical response curve of nonlinear self-excited oscillator to forced oscillations, Eq. (3).

amplitudes. In addition, the harmonic associated with the instability at 922 Hz also disappears. Because of this reduction in the instability mode and its harmonic, the acoustic power in the 0–1000 Hz range is significantly reduced. The maximum reduction in acoustic power in the spectra between 0 and 1000 Hz is 90% for this case.

The typical dependence of the natural instability amplitude on the driving amplitude is shown in Fig. 2b. At the highest driving amplitudes, the instability has essentially disappeared at the cost of the increase in amplitude at the driven frequency. There are several basic features of the instability amplitude dependence upon driving amplitude that can be discerned from Fig. 2b. First, the instability amplitude is independent of the forcing amplitude for some amplitude range A_L before decreasing. Second, the instability amplitude decreases with some slope δ_p for further driving amplitude increases. Third, the instability amplitude essentially stops changing in value (usually near zero values) with increasing disturbance amplitudes above some entrainment driving amplitude A_e. For example, in Fig. 2b, the entrainment amplitude perturbation velocity is ~21% of the mean velocity at the nozzle exit. Also shown in the figure is the overall rms pressure amplitude of the driven oscillation and natural mode, which has a minimum near the entrainment amplitude and then begins to rise with increased forcing levels, due to the growing amplitude of the imposed oscillations. Finally, the dependence of the unstable mode amplitude upon the driving amplitude exhibits some hysteresis, with typical levels on the order of $u'/u_0 = 0.03$.

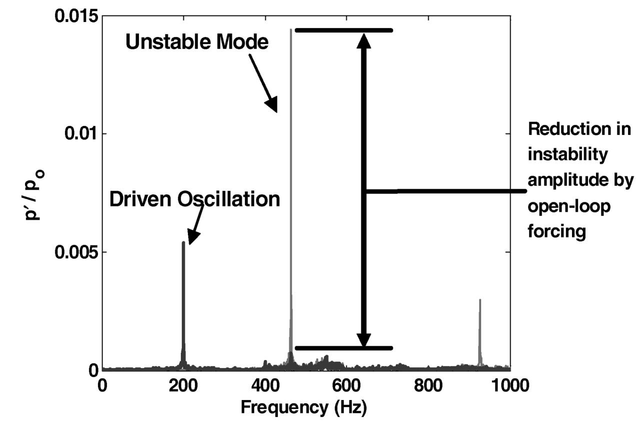
These characteristics depend significantly upon driving frequency, as shown in Fig. 3, which shows substantial and nonmonotonic variation in A_e. Although not shown, the entrainment amplitude based upon perturbation pressure, rather than velocity, exhibits a different trend. It increases monotonically with decreasing frequency down to about 230 Hz and then decreases for lower frequencies. This is due to the frequency dependence of the pressure–velocity relation, and is controlled by the overall system acoustics. As shown in Bellows [8], the slope of the instability amplitude roll off δ_p ranges from 0.05–0.2, whereas A_L ranges from $u'/u_0 = 0.02$–0.10, depending upon frequency.

As these results have direct implication on open-loop forcing as an active control methodology, it is of interest to analyze the total acoustic power reduction in the 0–1000 Hz range, where power is defined as

$$\text{power} = \int |p'|^2 \mathrm{d}f \tag{4}$$

Table 1 Combustor operating conditions

	Condition 1	Condition 2
Equivalence ratio, ϕ	0.83	0.83
Natural frequency, Hz, ±5 Hz	461 Hz	350 Hz
Flow velocity, m/s	22 m/s	22 m/s
Pressure instability amplitude, p'/p_0	0.013–0.015	0.02–0.024



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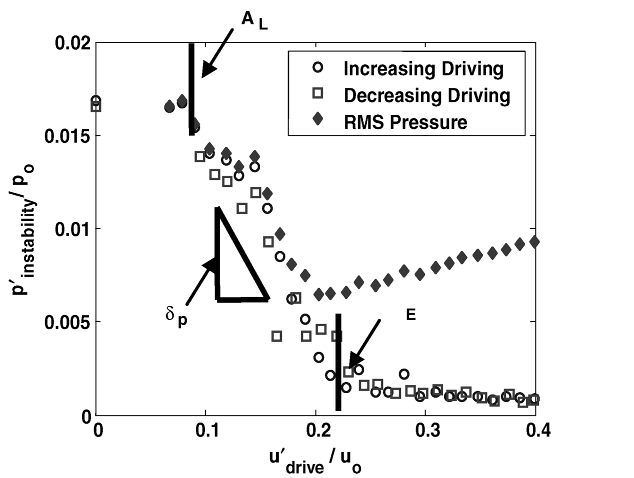


b)

Fig. 2 Effect of external oscillations upon self-excited instability, as reflected in a) spectrum of combustor pressure at two driving amplitudes, $f_{\text{ins}} = 461$ Hz, $f_{\text{drive}} = 200$ Hz, and b) instability amplitude dependence upon driving velocity amplitude at $f_{\text{drive}} = 200$ Hz.

This reduction in acoustic power is quantified by

$$\text{power reduction} = \frac{\text{power}_0 - \text{power}_{\min}}{\text{power}_0} \tag{5}$$

where power_0 and $\text{power}_{\min}$ represent the acoustic power in the absence of driving and the minimum acoustic power achieved over the velocity amplitude range. This maximum reduction in acoustic pressure power usually occurs at a forcing amplitude close to A_e. The acoustic power is reduced by at least 70% and up to 90% for condition 1.

We next consider condition 2, corresponding to a lower instability frequency, $f_{\text{ins}} = 350$ Hz (obtained with the longer tube), at the same conditions. The instability amplitude is about 50% higher than condition 1, about 2.2% of the mean pressure. The forcing frequencies investigated ranged from 150 to 490 Hz.

Figure 4a plots the maximum percentage reduction in acoustic power in the 0–1000 Hz range that can be achieved. Note that, at this operating condition, the effect of driving does not reduce the instability amplitude to near the extent it did for condition 1. Although the reasons for these differences in the effects of open-loop forcing are not understood, two factors may be significant: 1) larger levels of acoustic forcing required to overcome the initially higher amplitude oscillations, and 2) these higher driving amplitudes, besides acting to quench the instability, introduce other nonlinear phenomenon, such as frequency shifts and spectral broadening.

This latter point is clearly shown in Fig. 4b, which indicates that the instability shifts from 350 to 390 Hz. This figure plots two pressure spectra for a driving frequency of 260 Hz, one corresponding to the unforced condition and the other at a driving

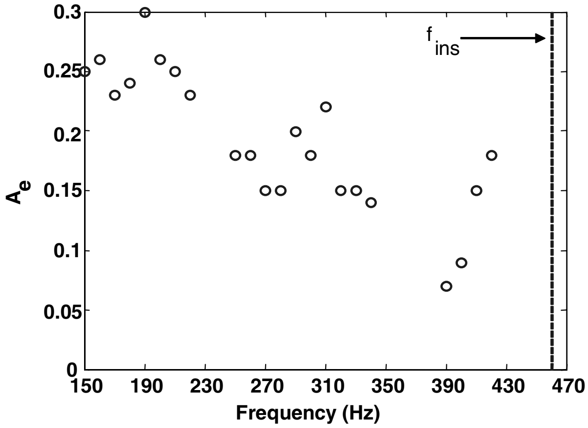
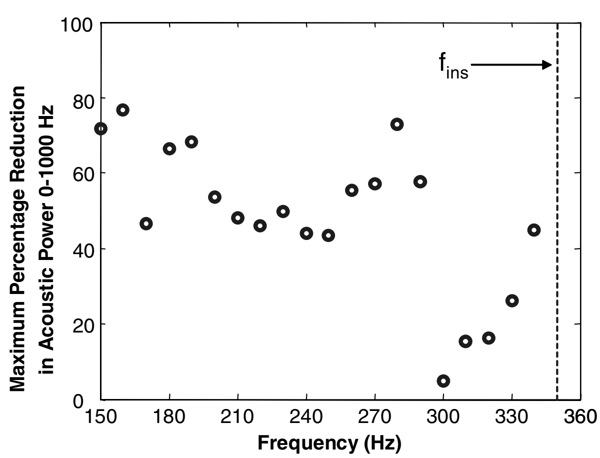


Fig. 3 Dependence of velocity entrainment amplitude A_E upon driving frequency, $f_{ins} = 461$ Hz.

velocity amplitude of $u'_{drive}/u_0 = 0.25$, where entrainment was seen to occur at many of the frequencies investigated in condition 1. For the no-driving case, the pressure spectrum shows the oscillations at the fundamental, 350 Hz, and slightly at the first harmonic around 700 Hz. As acoustic forcing is added, the instability amplitude at 350 Hz decreases somewhat, and shifts to a higher frequency, 390 Hz. At this driving frequency (260 Hz), the peak seen near 380–390 Hz does not readily correspond to any harmonics of the fundamental frequency of the driving signal, nor of the natural mode in the combustor.



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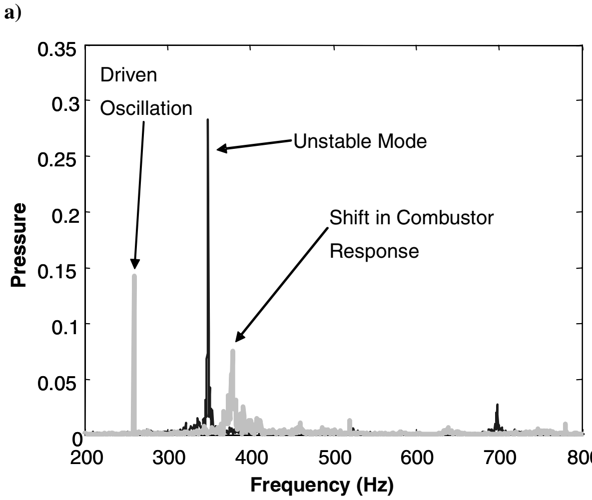
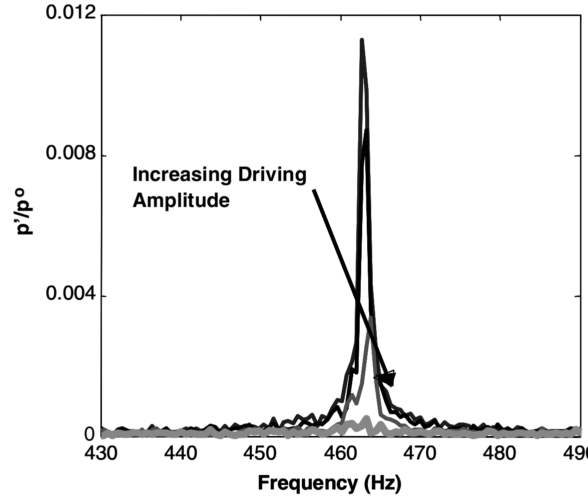


b)

Fig. 4 Effect of external oscillations upon acoustic pressure a) power reduction due to external forcing, $f_{ins} = 350$ Hz, and b) spectrum as driving amplitude is increased, $f_{ins} = 350$ Hz, $f_{drive} = 260$ Hz.



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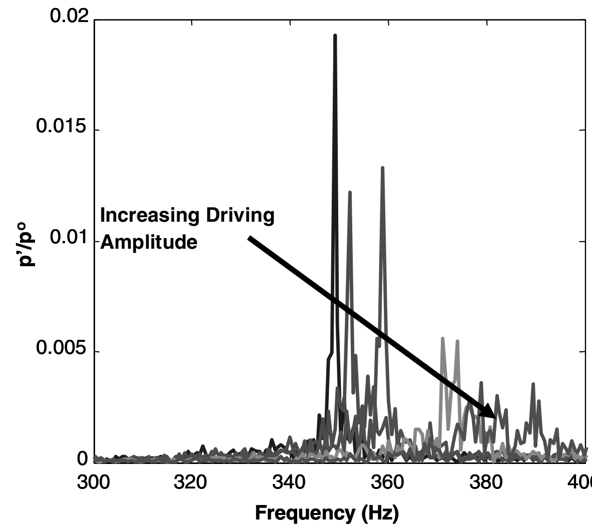

b)

Fig. 5 Dependence of pressure spectra in vicinity of nominal instability frequency for a) condition 1 ($f_{ins} = 461$ Hz, $f_{drive} = 200$ Hz), b) condition 2 ($f_{ins} = 350$ Hz, $f_{drive} = 260$ Hz).

This behavior is illustrated further in Fig. 5. Figure 5b plots the pressure spectra in the frequency range of 340–400 Hz for a driving frequency of 260 Hz. For increasing driving amplitudes, the pressure spectra clearly shows the transition of the instability to higher frequencies, away from the nominal value of 350 Hz. In addition, the frequency shifted band is also clearly broadened in frequency space. In contrast, Fig. 5a plots similar spectra for condition 1 between 450 and 500 Hz for increasing driving velocity amplitudes for a driving frequency of 200 Hz. In this case, it is clear that the instability is quenched and no such shift in the frequency is present. This difference, although not understood, is a distinguishing feature between cases in which open-loop forcing was and was not effective. Furthermore, the instability amplitude, even when shifted, is never reduced to near zero values with increasing driving amplitude.

Conclusions

This study shows that external forcing introduces several effects on the self-excited oscillations by altering their amplitude, spectral bandwidth, and/or frequency. The effectiveness of open-loop control of the combustor significantly depends on the operating condition. For cases in which the instability mode is reduced slightly and shifts in frequency, quenching does not occur, although some amplitude reductions may be observed. In addition, for all conditions investigated, the ability to significantly affect the self-excited mode is quite difficult at driven frequencies above the instability frequency. Only at a few select cases has open-loop forcing been able to achieve this quenched state.

No analysis of the actual flame and flow response under the influence of self-excited and driven oscillations has been performed, but is clearly critical to understand these processes. The receptivity of the flow to excitation and the various natural hydrodynamic instabilities will clearly have a strong frequency/amplitude dependence [10]. Similarly, the dynamics of the flame could also play a very large role in the quenching/entrainment behavior of the system response.

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